Matematica în Bucovina. Concursul Internaţional de matematică „Memorialul *David Hrimiuc*” ediţia a XIII - a, 4 noiembrie – 6 noiembrie 2016

Clasa a VII - a

Barem de corectare

1. (7p) Dacă a , b , $c>0$ cu $a\cdot b\cdot c=1$, calculați $E=\frac{2015+a}{1+a+ab}+\frac{2015+b}{1+b+bc}+\frac{2015+c}{1+c+ac}$.

Soluție.

$$E=\frac{2015+a}{1+a+ab}+\frac{2015+b}{1+b+bc}+\frac{2015+c}{1+c+ac}\overset{(2p)}{=}\frac{2015+a}{1+a+ab}+\frac{a\cdot(2015+b)}{a\cdot(1+b+bc)}+\frac{ab\cdot(2015+c)}{ab\cdot(1+c+ac)}=$$

$$\overset{(1p)}{=}\frac{2015+a}{1+a+ab}+\frac{2015a+ab}{a+ab+abc}+\frac{2015\cdot ab+abc}{ab+abc+a\cdot abc}\overset{(2p)}{=}\frac{2015+a}{1+a+ab}+\frac{2015a+ab}{a+ab+1}+\frac{2015\cdot ab+1}{ab+1+a}=$$

$$\overset{(1p)}{=}\frac{2015\cdot(1+a+ab)+a+ab+1}{1+a+ab}\overset{(1p)}{=}2016$$

2. a) (2p) Arătați că $a^2+a+\frac{1}{4}$ este pătrat perfect pentru orice a real.

Soluție. $a^2+a+\frac{1}{4}\overset{(0,5p)}{=}a^2+\frac{1}{2}a+\frac{1}{2}a+\frac{1}{4}\overset{(0,5p)}{=}a\left(a+\frac{1}{2}\right)+\frac{1}{2}\left(a+\frac{1}{2}\right)=$

$$\overset{(0,5p)}{=}\left(a+\frac{1}{2}\right)\left(a+\frac{1}{2}\right)\overset{(0,5p)}{=}\left(a+\frac{1}{2}\right)^2$$

b) (5p) Determinați numerele reale x și y pentru care $max\left\{x^2+y+\frac{1}{4},y^2+x+\frac{1}{4}\right\}\le 0$, unde $max\{a,b\}$ reprezintă cel mai mare număr dintre a și b.

Soluție.

$$max\left\{x^2+y+\frac{1}{4},y^2+x+\frac{1}{4}\right\}\le 0\overset{(1p)}{\Rightarrow}\begin{cases}x^2+y+\frac{1}{4}\le 0\\ y^2+x+\frac{1}{4}\le 0\end{cases}\overset{(1p)}{\Rightarrow}x^2+y+\frac{1}{4}+y^2+x+\frac{1}{4}\le 0\Rightarrow$$

$$\overset{(0,5p)}{\Rightarrow}x^2+x+\frac{1}{4}+y^2+y+\frac{1}{4}\le 0\overset{(1p)}{\Rightarrow}\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2\le 0\overset{(0,5p)}{\Rightarrow}\begin{cases}\left(x+\frac{1}{2}\right)^2=0\\ \left(y+\frac{1}{2}\right)^2=0\end{cases}\overset{(0,5p)}{\Rightarrow}x=y=-\frac{1}{2}$$

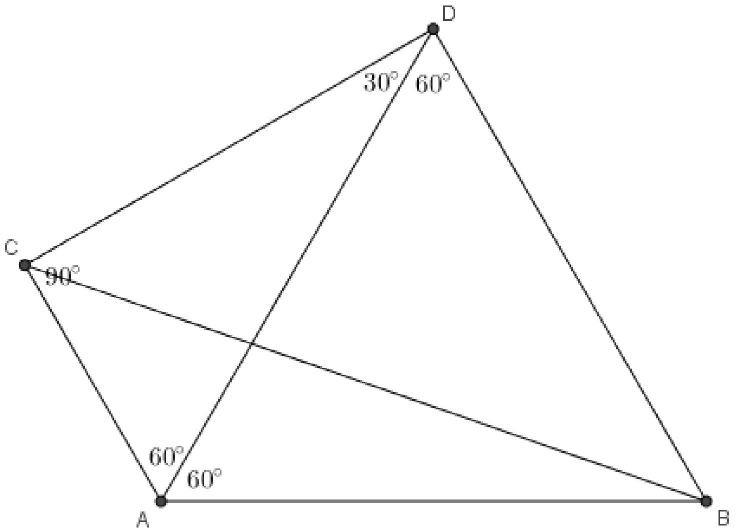
Verifică $x=y=-\frac{1}{2}$ *este sol.* $(0,5p)$.

3. (7p) În triunghiul ABC avem $AB=2\cdot AC$ şi $m(\angle BAC)=120^\circ$. Bisectoarea unghiului BAC intersectează perpendiculara în C pe AC în D. Arătaţi că $AC\parallel BD$.

Florin Mihai Antoche, Galaţi
(E:14963, GMB 2 / 2016)

Soluţie.

(0,5p)



$$\left.\begin{array}{r}\left.\begin{array}{r}\left.\begin{array}{r}\left.\begin{array}{r}\left.\begin{array}{l}m(\angle BAC)=120^0\\ [AD-bis.\angle BAC\end{array}\right\}\overset{(1p)}{\Rightarrow} m(\angle DAC)=60^0\\ DC\perp AC\end{array}\right\}\overset{(0,5p)}{\Rightarrow} m(\angle ADC)=30^0\\ ACD-\text{dreptunghic în } C\end{array}\right\}\overset{(1p)}{\Rightarrow} AD=2\cdot AC\\ AB=2\cdot AC\end{array}\right\}\overset{(0,5p)}{\Rightarrow} AD=AB\\ m(\angle BAD)=60^0\end{array}\right\}\Rightarrow$$

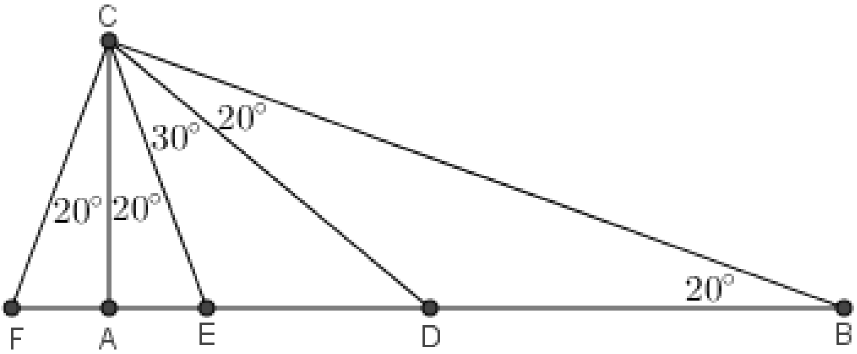
$$\overset{(1p)}{\Rightarrow} ABD-\text{echilateral}\overset{(0,5p)}{\Rightarrow}\left.\left.\begin{array}{r}\left.\begin{array}{r}m(\angle ADB)=60^0\\ m(\angle ADC)=30^0\end{array}\right\}\overset{(0,5p)}{\Rightarrow} m(\angle BDC)=90^0\overset{(0,5p)}{\Rightarrow} DB\perp CD\\ AC\perp CD\end{array}\right.\right\}\overset{(1p)}{\Rightarrow} AC\parallel BD.$$

4. (7p) Fie ABC un triunghi dreptunghic cu ipotenuza BC şi $m(\angle B)=20^\circ$. Pe latura $[AB]$ considerăm punctele D şi E astfel încât $m(\angle ACD)=m(\angle BCE)=50^\circ$. Arătaţi că $BE=2AD$.

Costel Anghel, Negreni, Scorniceşti
(S:E16.26, Supliment GMB 1 / 2016)

Soluţie.

(0,5p)



$$\left.\begin{array}{r}\left.\begin{array}{r}ABC-\text{dreptunghic în } A\\ m(\angle B)=20^0\end{array}\right\}\overset{(0,5p)}{\Rightarrow} m(\angle ACB)=70^0\\ m(\angle ACD)=m(\angle BCE)=50^0\end{array}\right\}\overset{(1p)}{\Rightarrow} m(\angle BCD)=m(\angle ACE)=20^0,\ m(\angle DCE)=30^0$$

$$\left.\begin{array}{r} \left.\begin{array}{r} \left.\begin{array}{l} Fie\ F\in AB\ a.\hat{i}.\ [FA]\equiv[AE] \\ CA\perp AB \end{array}\right\} \overset{(1p)}{\Rightarrow} m(\sphericalangle FCA)=m(\sphericalangle ACE)=20^0 \\ m(\sphericalangle ACB)=70^0 \end{array}\right\} \overset{(0,5p)}{\Rightarrow} m(\sphericalangle FCB)=90^0 \\ m(\sphericalangle B)=20^0 \end{array}\right\} \overset{(0,5p)}{\Rightarrow} m(\sphericalangle CFD)=70^0 \quad (1)$$

$$\left.\begin{array}{r} m(\sphericalangle FCD)=20^0+20^0+30^0 \overset{(0,5p)}{=} 70^0 \overset{(1)}{\Rightarrow} \sphericalangle FCD\equiv\sphericalangle CFD \overset{(0,5p)}{\Rightarrow} (CD)\equiv(FD) \\ m(\sphericalangle DBC)=m(\sphericalangle BCD)=20^0 \overset{(0,5p)}{\Rightarrow} (CD)\equiv(DB) \end{array}\right\} \overset{(0,5p)}{\Rightarrow} (FD)\equiv(DB) \Rightarrow$$

$$\Rightarrow BD=FE+ED \overset{(0,5p)}{\Rightarrow} BD=2\cdot AE+ED \Rightarrow BD+DE=2\cdot(AE+ED) \overset{(0,5p)}{\Rightarrow} BE=2\cdot AD.$$

Notă: **Orice altă rezolvare corectă se punctează corespunzător.**