Matematica în Bucovina. Concursul Internaţional de matematică „Memorialul David Hrimiuc”
ediţia a XII - a, 30 octombrie – 1 noiembrie 2015

Clasa a VIII- a

Barem de corectare

1. $\left.\begin{array}{r} x^2-25y^2-8=5^z \\ z<0 \Rightarrow 5^z \notin \mathbb{Z} \\ x,y\in\mathbb{Z} \Rightarrow x^2-25y^2-8\in\mathbb{Z} \end{array}\right\} fals! \Rightarrow z\in\mathbb{N}$ **(1p)**

$\left.\begin{array}{r} \left.\begin{array}{r} x^2-3=25y^2+5^z+5 \\ z\in\mathbb{N}^* \Rightarrow 5/25y^2+5^z+5 \end{array}\right\} \Rightarrow 5/x^2-3 \\ x\in\mathbb{Z} \Rightarrow u\left(x^2\right)\in\{0,1,4,5,6,9\} \end{array}\right\} fals! \Rightarrow z\notin\mathbb{N}^*$ **(1,5p)**

Rezultă $z=0$ **(0,5p)**

$x^2-25y^2-5^z=8 \overset{z=0}{\Leftrightarrow} x^2-25y^2=9 \Leftrightarrow (x-5y)(x+5y)=9 \Leftrightarrow$ **(1p)**

$\Leftrightarrow \begin{cases} x-5y=-9 \\ x+5y=-1 \end{cases} sau \begin{cases} x-5y=-3 \\ x+5y=-3 \end{cases} sau \begin{cases} x-5y=-1 \\ x+5y=-9 \end{cases} sau \begin{cases} x-5y=1 \\ x+5y=9 \end{cases} sau \begin{cases} x-5y=3 \\ x+5y=3 \end{cases} sau \begin{cases} x-5y=9 \\ x+5y=1 \end{cases} \Leftrightarrow$ **(1,5p)**

$\Leftrightarrow \begin{cases} x=-3 \\ y=0 \end{cases} sau \begin{cases} x=3 \\ y=0 \end{cases}$ **(1p)**

Deci, $\begin{cases} x=\pm 3 \\ y=0 \\ z=0 \end{cases}$. **(0,5p)**

2. a) Soluţia 1.

$\frac{2x+n}{n+2}\geq\frac{n+4}{3x+n+1}, (\forall)n,x\in\mathbb{N}^* \Leftrightarrow (2x+n)(3x+n+1)\geq(n+2)(n+4)\ (\forall)n,x\in\mathbb{N}^*$ **(1p)** $\Leftrightarrow$

$\Leftrightarrow 6x^2+(5n+2)x-(5n+8)\geq 0, (\forall)n,x\in\mathbb{N}^*$ **(1p)** $\Leftrightarrow (x-1)(6x+5n+8)\geq 0$, $(\forall)n,x\in\mathbb{N}^*$ ($\mathcal{A}$) **(1p)**

$\Rightarrow \frac{2x+n}{n+2}\geq\frac{n+4}{3x+n+1}, (\forall)n,x\in\mathbb{N}^*$. **(1p)** Egalitate d.d. $x=1$.

Soluţia 2.

$\frac{2x+n}{n+2}\geq 1 \Leftrightarrow 2x+n\geq n+2 \Leftrightarrow x\geq 1, (\forall)n,x\in\mathbb{N}^*$ ($\mathcal{A}$) $\Rightarrow \frac{2x+n}{n+2}\geq 1$, $(\forall)n,x\in\mathbb{N}^*$ **(1,5p)**

$\frac{n+4}{3x+n+1}\leq 1 \Leftrightarrow n+4\leq 3x+n+1 \Leftrightarrow x\geq 1$, $(\forall)n,x\in\mathbb{N}^*$ ($\mathcal{A}$) $\Rightarrow \frac{n+4}{3x+n+1}\leq 1, (\forall)n,x\in\mathbb{N}^*$ **(1,5p)**

Rezultă: $\frac{2x+n}{n+2}\geq\frac{n+4}{3x+n+1}, (\forall)n,x\in\mathbb{N}^*$ **(1p)** Egalitate d.d. $x=1$.

b) De la punctul **a)** rezultă: $\frac{2x+1}{3}+\frac{2x+2}{4}+\frac{2x+3}{5}+\dots+\frac{2x+1008}{1010}\geq 1008$, $(\forall)x\in\mathbb{N}^*$ **(1p)** și

$\frac{1013}{3x+1010}+\frac{1014}{3x+1011}+\frac{1015}{3x+1012}+\dots+\frac{2020}{3x+2017}\leq 1008, (\forall)\ x\in\mathbb{N}^*$ **(1p)**

Avem:

$\frac{2x+1}{3}+\frac{2x+2}{4}+\frac{2x+3}{5}+\dots+\frac{2x+1008}{1010}=\frac{1013}{3x+1010}+\frac{1014}{3x+1011}+\frac{1015}{3x+1012}+\dots+\frac{2020}{3x+2017}$, $x\in\mathbb{N}^* \Leftrightarrow$

$\Leftrightarrow \begin{cases} \frac{2x+1}{3}+\frac{2x+2}{4}+\frac{2x+3}{5}+\dots+\frac{2x+1008}{1010}=1008 \\ \frac{1013}{3x+1010}+\frac{1014}{3x+1011}+\frac{1015}{3x+1012}+\dots+\frac{2020}{3x+2017}=1008 \end{cases}$, $x\in\mathbb{N}^* \Leftrightarrow x=1$ **(1p)**

3. a) $\left(\sqrt{x}-8\right)^2 \geq 0$, $(\forall) x \geq 0$ **(0,5p)** $\Rightarrow x-16\sqrt{x}+64 \geq 0$, $(\forall) x \geq 0$ **(0,5p)**

$\Rightarrow 2(\sqrt{x}-4) \leq \frac{x}{8}$, $(\forall) x \geq 0$ **(0,5p)**

b) $|x|^2 = x^2$, $(\forall) x \in \mathbb{R}$ **(0,5p)** $\left(|x|-28\right)^2 \geq 0$, $(\forall) x \in \mathbb{R}$ **(0,5p)**

$\Rightarrow x^2-56|x|+784 \geq 0$, $(\forall) x \in \mathbb{R}$ **(0,5p)** $\Rightarrow 7(|x|-14) \leq \frac{x^2}{8}$, $(\forall) x \in \mathbb{R}$ **(1p)**

c) Soluţia 1. De la punctul **a)** rezultă, înlocuind pe x cu y: $\sqrt{y} \leq \frac{y+64}{16}$, $(\forall) y \geq 0$. **(1p)**

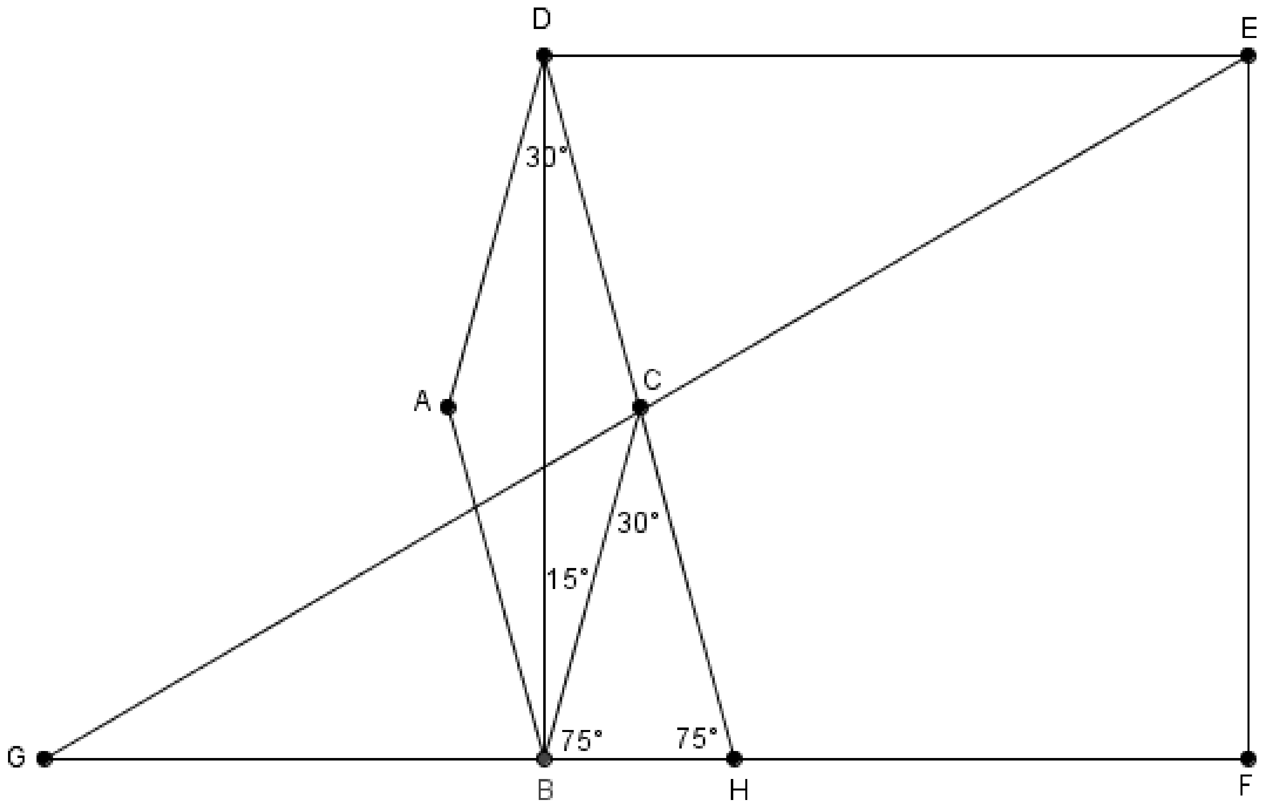
De la punctul **b)** rezultă, înlocuind pe x cu $\sqrt{x}$: $\sqrt{x} \leq \frac{x+784}{56}$, $(\forall) y \geq 0$ **(1p)**

Prin adunare, obţinem:

$$\sqrt{x}+\sqrt{y} \leq \frac{x+784}{56}+\frac{y+64}{16}=\frac{2x+7y+1568+448}{112} \leq \frac{2016+2016}{112}=36 \Rightarrow \sqrt{x}+\sqrt{y} \leq 36 \quad \textbf{(1p)}$$

Obs. $\sqrt{x} \leq \frac{x+a^2}{2a}$, $(\forall) x \geq 0$, $a > 0$.

4.



a) Figura **(1p)**

$$\left.\begin{array}{r} ACBD-romb \Rightarrow BC \parallel AD \Rightarrow \angle BCH \equiv \angle ADC \\ m(\angle ADC)=30^\circ \end{array}\right\} \Rightarrow m(\angle BCH)=30^\circ \quad (0,5p)$$

$$\left.\begin{array}{r} \left.\begin{array}{r} ACBD-romb \\ m(\angle ABC)=30^\circ \end{array}\right\} \Rightarrow m(\angle DBC)=15^\circ \\ ABCD-pătrat \Rightarrow m(\angle DBH)=90^\circ \end{array}\right\} \Rightarrow m(\angle CBH)=75^\circ \quad (0,5p)$$

$\Rightarrow$

$\Rightarrow m(\angle CHB)=180^\circ-30^\circ-75^\circ \quad (0,5p) \quad \Rightarrow \angle CBH \equiv \angle CHB \Rightarrow CBH-isoscel \quad (0,5p)$

b)

$$\left.\begin{array}{l}\left.\begin{array}{l}CBH-isoscel \Rightarrow [CB]\equiv[CH]\\ ABCD-romb \Rightarrow [CB]\equiv[DC]\end{array}\right\} \Rightarrow [CH]\equiv[DC] \quad (0,5p)\\ BDEF-pătrat \Rightarrow BH \parallel DE \Rightarrow \angle CDE \equiv \angle CHG \ (corep.) \quad (0,5p)\\ \angle DCE \equiv \angle HGC \quad (opuse\ la\ vârf)\end{array}\right\} \overset{(ULU)}{\Rightarrow} \Delta DCE \equiv \Delta HCG \quad (0,5p) \Rightarrow$$

$$\Rightarrow [CE]\equiv[CG] \Rightarrow C-mijl.\ [EG] \Rightarrow [FC]-mediană\ în\ EFG \qquad (0,5p)$$

c) Fie K în interiorul pătratului $BDEF$ a.î. EKF este triunghi echilateral.

$$\left.\begin{array}{l}EKF-echilat. \Rightarrow [EK]\equiv[FK]\equiv[EF]\\ BDEF-pătrat \Rightarrow [BF]\equiv[EF]\equiv[ED]\end{array}\right\} \Rightarrow [FK]\equiv[BF],\ [EK]\equiv[ED] \Rightarrow BKF,\ DKE-isoscele$$

$$\left.\begin{array}{l}\left.\begin{array}{l}EKF-echilat. \Rightarrow m(\angle EFK)=60^\circ\\ BDEF-pătrat \Rightarrow m(\angle EFB)=90^\circ\end{array}\right\} \Rightarrow m(\angle KFB)=30^\circ\\ \qquad [FK]\equiv[BF]\end{array}\right\} \Rightarrow m(\angle FBK)=75^\circ \Rightarrow m(\angle DBK)=15^\circ \quad (0,5p)$$

$$\left.\begin{array}{l}\left.\begin{array}{l}EKF-echilat. \Rightarrow m(\angle FEK)=60^\circ\\ BDEF-pătrat \Rightarrow m(\angle FED)=90^\circ\end{array}\right\} \Rightarrow m(\angle KED)=30^\circ\\ \qquad [EK]\equiv[ED]\end{array}\right\} \Rightarrow m(\angle EDK)=75^\circ \Rightarrow m(\angle BDK)=15^\circ \quad (0,5p)$$

$$\left.\begin{array}{l}m(\angle BDK)=m(\angle BDC)=15^\circ \Rightarrow [DC=[DK\\ m(\angle DBK)=m(\angle DBC)=15^\circ \Rightarrow [BC=[BK\end{array}\right\} \Rightarrow K=C \Rightarrow m(\angle DCE)=75^\circ \quad (0,5p) \Rightarrow$$

$$\left.\begin{array}{l}\Rightarrow m(\angle GCH)=75^\circ\\ dar,\ m(\angle CHG)=75^\circ\end{array}\right\} \Rightarrow CGH-isoscel \qquad (0,5p)$$

Notă: Orice altă rezolvare corectă se punctează corespunzător.