Matematica în Bucovina. Concursul Internaţional de matematică „Memorialul David Hrimiuc”
ediţia a XII - a, 30 octombrie – 1 noiembrie 2015

Clasa a VII- a

Barem de corectare

1. Fie $A=7^{2015}-6^{2015}$. Avem:

$A=7^{2015}-6^{2015}=\left(6+1\right)^{2015}-6^{2015}=M_6+1-M_6=M_6+1\Rightarrow 6/A-1$ **(2,5p)**

$A=7^{2015}-6^{2015}=7^{2015}-\left(7-1\right)^{2015}=M_7-\left(M_7-1\right)=M_7+1\Rightarrow 7/A-1$ **(2,5p)**

$$\left.\begin{array}{l}6/A-1\\ 7/A-1\\ \left(6,7\right)=1\end{array}\right\}\Rightarrow 42/A-1 \quad (1p) \quad \Rightarrow A-1=42k,\ k\in\mathbb{N}\Rightarrow A=42k+1\Rightarrow r=1 \quad (1p)$$

2. Soluţia 1.

$$\frac{2x^2+15}{3x+2}=\frac{1}{9}\cdot\frac{18x^2+135}{3x+2}=\frac{1}{9}\cdot\frac{18x^2+12x-12x-8+143}{3x+2}= \quad (1p)$$

$$=\frac{1}{9}\cdot\frac{6x\left(3x+2\right)-4\left(3x+2\right)+143}{3x+2} \quad (1p)=\frac{1}{9}\left(6x-4+\frac{143}{3x+2}\right) \quad (1p)$$

$$\frac{2x^2+15}{3x+2}\in\mathbb{N}\Rightarrow 6x-4-\frac{143}{3x+2}\in\mathbb{N}\Rightarrow\frac{143}{3x+2}\in\mathbb{N} \quad (1p) \quad \Rightarrow 3x+2/143 \quad (1p)\Rightarrow$$

$$\overset{x\in\mathbb{N}}{\Rightarrow} 3x+2=1 \ sau\ 3x+2=11\ \ sau\ 3x+2=13\Rightarrow x=3 \quad (1p)$$

3 este soluţie, pentru că: $\dfrac{2\cdot 3^2+15}{3\cdot 3+2}=\dfrac{33}{11}=3\in\mathbb{N}$ **(1p)**

Soluţia 2.

$$\left.\left.\begin{array}{r}\dfrac{2x^2+15}{3x+2}\in\mathbb{N}\Rightarrow 3x+2/2x^2+15\Rightarrow 3x+2/6x^2+45\ \ (1p)\\ 2/3x+2\Rightarrow 3x+2/6x^2+4x\ \ (1p)\end{array}\right\}\Rightarrow 3x+2/4x-45\Rightarrow 3x+2/12x-135\ \ (1p) \atop 3x+2/3x+2\Rightarrow 3x+2/12x+8\ \ (1p)\right\}\Rightarrow$$

$$\Rightarrow 3x+2/143 \quad (1p) \quad \overset{x\in\mathbb{N}}{\Rightarrow} \quad 3x+2=1\ sau\ 3x+2=11\ \ sau\ 3x+2=13\Rightarrow x=3 \quad (1p)$$

3 este soluţie, pentru că: $\dfrac{2\cdot 3^2+15}{3\cdot 3+2}=\dfrac{33}{11}=3\in\mathbb{N}$ **(1p)**

3. a)

$$\frac{3x^2}{x^2+xy+y^2}\ge 2-\frac{y}{x}\ \left|\cdot x\left(x^2+xy+y^2\right)>0\Leftrightarrow 3x^3\ge 2x^3+2x^2y+2xy^2-x^2y-xy^2-y^3 \quad (1p)\Leftrightarrow\right.$$

$$\Leftrightarrow x^3-x^2y-xy^2+y^3\ge 0 \quad (0,5p) \quad \Leftrightarrow\left(x+y\right)\left(x^2-xy+y^2\right)-xy\left(x+y\right)\ge 0 \quad (0,5p)\Leftrightarrow$$

$$\Leftrightarrow\left(x+y\right)\left(x-y\right)^2\ge 0 \quad adev. \quad (0,5p)\Rightarrow\frac{3x^2}{x^2+xy+y^2}\ge 2-\frac{y}{x} \quad (0,5p)$$

Obs. Egalitatea are loc d.d. $x=y$.

b)
$$\left.\begin{array}{ll}\dfrac{3x^2}{x^2+xy+y^2}\geq 2-\dfrac{y}{x}\Rightarrow \dfrac{x^2}{y\left(x^2+xy+y^2\right)}\geq \dfrac{2x-y}{3xy} & (0,5p)\\ \dfrac{3y^2}{y^2+yz+z^2}\geq 2-\dfrac{z}{y}\Rightarrow \dfrac{y^2}{z\left(y^2+yz+z^2\right)}\geq \dfrac{2y-z}{3yz} & (0,5p)\\ \dfrac{3z^2}{z^2+xz+x^2}\geq 2-\dfrac{x}{z}\Rightarrow \dfrac{z^2}{x\left(z^2+xz+x^2\right)}\geq \dfrac{2z-x}{3xz} & (0,5p)\end{array}\right\}\Rightarrow$$

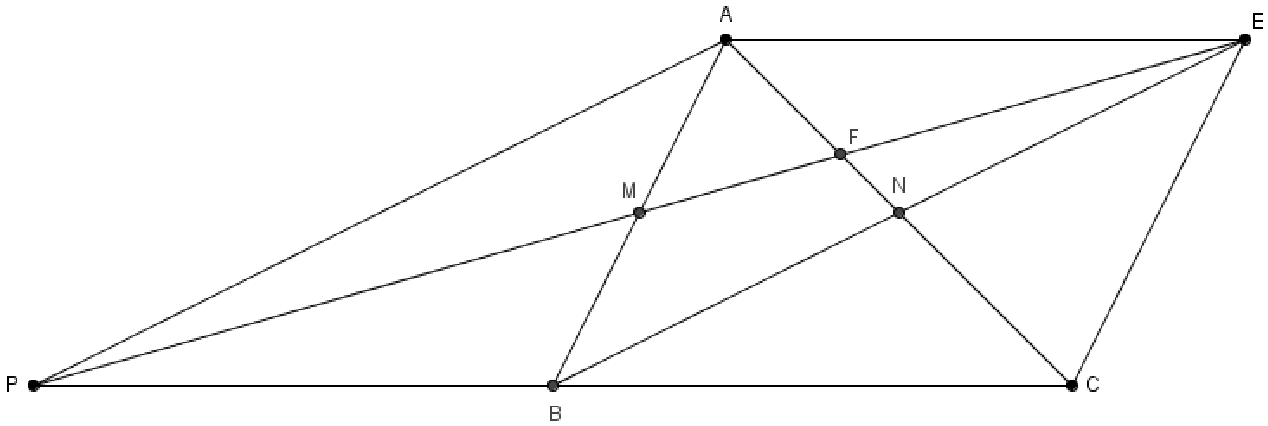
$$\Rightarrow \frac{x^2}{y\left(x^2+xy+y^2\right)}+\frac{y^2}{z\left(y^2+yz+z^2\right)}+\frac{z^2}{x\left(z^2+xz+x^2\right)}\geq \frac{2x-y}{3xy}+\frac{2y-z}{3yz}+\frac{2z-x}{3xz}= \qquad (0,5p)$$

$$=\frac{2xz-yz+2xy-xz+2yz-xy}{3xyz}=\frac{xz+yz+xy}{3xyz}\quad (0,5p)=\frac{1}{3}\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\frac{1}{3}\cdot 6045=2015\quad (0,5p)$$

Egalitatea are loc dacă şi numai dacă $x=y=z$ (din primele 3 relaţii), adică:

$x=y=z=\dfrac{1}{2015}$ (din condiţia suplimentară impusă).

4. Figura **(1p)**



$$\left.\begin{array}{r}N-mijl.\left[AC\right]\\ E=sim_N B\Rightarrow N-mijl.\left[BE\right]\quad (0,5p)\end{array}\right\}\Rightarrow ABCE-paralel.\Rightarrow AE\parallel BC\ ,\ \left[AE\right]\equiv\left[BC\right]\quad (0,5p)$$
$$\left.\begin{array}{r}\ldots\ \left[AE\right]\equiv\left[BC\right]\quad (0,5p)\\ P=sim_B C\Rightarrow\left[PB\right]\equiv\left[BC\right]\quad (0,5p)\end{array}\right\}\Rightarrow$$

$$\Rightarrow AE\parallel PB\ ,\ \left[AE\right]\equiv\left[PB\right]\quad (0,5p)\Rightarrow APBE-paralel.\quad (0,5p)\Rightarrow$$
$$\Rightarrow M-mijl.\left[AB\right]\quad (0,5p),\ \left[PM\right]\equiv\left[ME\right]\quad (0,5p)\quad (*)$$

$$\left.\begin{array}{l}M-mijl.\left[AB\right]\Rightarrow\left[EM\right]-med.\ \text{în}\ ABE\quad (0,5p)\\ N-mijl.\left[BE\right]\Rightarrow\left[AN\right]-med.\ \text{în}\ ABE\quad (0,5p)\end{array}\right\}\Rightarrow F-c.g.\ \text{în}\ ABE\quad (0,5p)\Rightarrow$$

$$\Rightarrow \frac{FM}{ME}=\frac{1}{3}\quad (0,5p)\overset{(*)}{\Rightarrow}\frac{FM}{MP}=\frac{1}{3}\quad (0,5p)$$

Notă: Orice altă rezolvare corectă se punctează corespunzător.